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Determining see-saw parameters from weak scale measurements?

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Abstract: The see-saw mechanism is a very attractive explanation for small neutrino masses, parametrized at the GUT scale by the right-handed Majorana mass matrix, $\mathcal{M}$, and the neutrino Yukawa matrix, $\mathbf{Y}_\nu$. We show that in a SUSY model with universal soft terms, $\mathcal{M}$ and $\mathbf{Y}_\nu$ can be calculated from the light neutrino masses, the MNS matrix, and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, which enters into the left-handed slepton radiative corrections. This suggests that *in principle* the GUT-scale inputs of the seesaw could be reconstructed from the neutrino and sneutrino mass matrices. We briefly discuss why this is impractical, but advocate the neutrino and sneutrino mass matrices as an alternative bottom-up parametrization of the seesaw.

Keywords: Beyond Standard Model, Supersymmetric Standard Model, Neutrino Physics.

Contents

1. Introduction and notation

The observed atmospheric [1, 2, 3] and solar [4] neutrino deficits suggest that neutrinos have small but non-zero masses. These can be elegantly explained via the see-saw [5] mechanism, where the left-handed neutrinos of the Standard Model get a small mass from their small mixing with the heavy right-handed singlet neutrinos. Unfortunately, the new physics appears at a very high energy scale, not directly accessible to experiment, which makes it difficult to constrain the see-saw parameters. Motivated by this, we will construct an alternative parameter space, in terms of quantities measurable (in principle) at low energies, where the experimental constraints can be readily imposed.

We consider the supersymmetric see-saw for two reasons: first, supersymmetry stabilizes the Higgs mass against the dangerous quadratic divergences that appear due to the presence of heavy particles (and in the context of the see-saw mechanism, we know that there are at least three heavy particles, namely, the right-handed neutrinos). Second, the presence of sleptons in the spectrum of the theory is crucial to our derivation, because one of our low-energy inputs is the slepton mass matrix.

The leptonic part of the superpotential reads

$$W_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H_1 + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H_2 - \frac{1}{2} {\nu_R^c}^T \mathcal{M} \nu_R^c \,, \tag{1.1}$$

where L_i and e_{Ri} $(i = e, \mu, \tau)$ are the left-handed lepton doublet and the right-handed charged-lepton singlet, respectively, and H_1 (H_2) is the hypercharge $-1/2$ $(+1/2)$ Higgs doublet. $\mathbf{Y_e}$ and $\mathbf{Y}_\nu$ are the Yukawa couplings that give masses to the charged leptons and generate the neutrino Dirac mass, and $\mathcal{M}$ is a 3×3 Majorana mass matrix that does not break the SM gauge symmetry. We do not make any assumptions about the structure of the matrices in eq. (1.1), but consider the most general case. Then, it can be proved that the number of independent physical parameters is 21: 15 real parameters and 6 complex phases [6].

It is natural to assume that the overall scale of $\mathcal{M}$, denoted by M, is much larger than the electroweak scale or any soft mass. Therefore, at low energies the right-handed neutrinos are decoupled and the corresponding effective lagrangian reads

$$\delta\mathcal{L}_{lep} = e_R^{c\,T}\mathbf{Y_e}L\cdot H_1 - \frac{1}{2}(\mathbf{Y}_\nu L\cdot H_2)^T\mathcal{M}^{-1}(\mathbf{Y}_\nu L\cdot H_2) + \text{h.c.}\,. \tag{1.2}$$

After the electroweak symmetry breaking, the left-handed neutrinos acquire a Majorana mass, given by

$$\mathcal{M}_\nu = \mathbf{m_D}^T\mathcal{M}^{-1}\mathbf{m_D} = \mathbf{Y}_\nu{}^T\mathcal{M}^{-1}\mathbf{Y}_\nu\langle H_2^0\rangle^2\,, \tag{1.3}$$

suppressed with respect to the typical fermion masses by the inverse power of the large scale M. In what follows, it will be convenient to extract the Higgs VEV by defining

$$\kappa = \mathcal{M}_\nu/\langle H_2^0\rangle^2 = \mathbf{Y}_\nu{}^T\mathcal{M}^{-1}\mathbf{Y}_\nu\,, \tag{1.4}$$

where $\langle H_2^0\rangle^2 = v_2^2 = v^2\sin^2\beta$ and $v = 174$ GeV. Working in the flavour basis in which the charged-lepton Yukawa matrix, $\mathbf{Y_e}$, and the gauge interactions are flavour-diagonal, the κ matrix can be diagonalized by the MNS [7] matrix U according to

$$U^T\kappa U = \text{diag}(\kappa_1,\kappa_2,\kappa_3) \equiv D_\kappa\,, \tag{1.5}$$

where U is a unitary matrix that relates flavour to mass eigenstates

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}\,, \tag{1.6}$$

and the κ_i can be chosen real and positive. U can be written as

$$U = V\cdot\text{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1)\,, \tag{1.7}$$

where ϕ and ϕ' are CP violating phases (if different from 0 or π) and V has the ordinary form of the CKM matrix

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta}\\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13}\\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}\end{pmatrix}\,. \tag{1.8}$$

The κ matrix, eq. (1.4), is at the moment our only experimental hint about the high energy physics that generates the neutrino masses. Unfortunately, it is not enough to reconstruct the whole theory. However, there is a second window onto the high energy physics, apart from the neutrino mass matrix: radiative corrections. Between the GUT scale and the Majorana mass scale, M, neutrino Yukawa couplings affect the renormalization of the slepton mass matrix through the combination $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$. These contributions can leave signatures at low energies and thus provide additional

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information about the theory at high energies. With certain assumptions, the information provided by radiative corrections is complementary to that provided by κ, so that *in principle* it could be possible to reconstruct the complete high energy theory. This is the case if the soft masses are universal at the high scale M_X, and if the most significant contributions to the slepton RGEs come from the supersymmetric SM with right-handed neutrinos.

There are many papers constraining the seesaw parameter space, so we would like to situate our work among this literature. Present measurements of κ do not determine the matrices $\mathbf{Y}_\nu$ and $\mathcal{M}$, so seesaw analyses can be categorized by what they use as inputs. A common approach is to start at the GUT scale, choose theoretically motivated textures for $\mathcal{M}$ and the Yukawa matrices, require that they induce a neutrino mass matrix consistent with observations, and then study other low energy predictions of the chosen texture. For recent top-down discussions, see for instance [8] (about leptogenesis), [9] (lepton flavour violation) and references therein. A more "bottom-up" approach [10], is to start from the experimental data on neutrino masses and mixings, and find some particular forms for $\mathbf{Y}_\nu$ and $\mathcal{M}$ that reproduce the neutrino data. Some theoretical input is required at the GUT scale, because there are fewer parameters in κ than there are in $\mathbf{Y}_\nu$ and $\mathcal{M}$. One can then study other low energy predictions of the chosen $\mathbf{Y}_\nu$ and $\mathcal{M}$. Finally, in [11] the authors found a parametrization of all the Yukawa couplings compatible with the low energy data, in terms of the masses of the right-handed neutrinos and some unknown parameters. This approach can be considered as "hybrid", since the parameter space is described both by high energy and low energy quantities. The bottom-up approach to the seesaw which we follow, is to start from present and hypothetical future observations (of Supersymmetry), and try to reconstruct $\mathcal{M}$ and $\mathbf{Y}_\nu$. All of our inputs will therefore be at the weak scale.

The paper is organized as follows. In section 2, we present the general procedure to reconstruct the high energy theory from the low energy inputs. It is evident that the more constrained the low energy parameters are, the better we know the high energy parameter space. Therefore, some words on the present and future constraints on the low energy observables are in order. This is done in section 3. Finally, in section 4, we discuss our results and some applications of our procedure.

2. General procedure

We take as our weak scale inputs the neutrino mass matrix, related to κ, and

$$P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \,. \tag{2.1}$$

At this stage, we do not have much information about P and we prefer to interpret it as a way to parametrize our ignorance of the high energy physics. However, this

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parametrization will turn out to be very convenient, because P is the combination of Yukawa couplings which enters in the slepton Renormalization Group Equations (RGEs).

Now we turn to the determination of $\mathbf{Y}_\nu$ and $\mathcal{M}$ from κ and P. We can always choose to work in the basis where the charged lepton mass matrix is diagonal. We can also choose to work in a basis of right-handed neutrinos where $\mathcal{M}$ is diagonal

$$\mathcal{M} = \mathrm{diag}(\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3) \equiv D_{\mathcal{M}}\,, \tag{2.2}$$

with $\mathcal{M}_i \geq 0$. In this basis, the neutrino Yukawa matrix must be necessarily non-diagonal, but can always be diagonalized by two unitary transformations:

$$\mathbf{Y}_\nu = V_R^\dagger D_Y V_L. \tag{2.3}$$

V_L and D_Y can be determined from P, since, using eq. (2.3),

$$P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = V_L^\dagger D_Y^2 V_L\,. \tag{2.4}$$

On the other hand, from $\kappa = \mathbf{Y}_\nu^T D_{\mathcal{M}}^{-1} \mathbf{Y}_\nu$ and eq. (2.3),

$$D_Y^{-1} V_L^* \kappa V_L^\dagger D_Y^{-1} = V_R^* D_{\mathcal{M}}^{-1} V_R^\dagger\,, \tag{2.5}$$

where the left hand side of this equation is known (κ is one of our inputs, and V_L and D_Y were obtained from eq. (2.4)). Therefore, V_R and $D_{\mathcal{M}}$ can also be determined. This shows that, working in the basis where the charged lepton Yukawa coupling, $\mathbf{Y_e}$, the right-handed Majorana mass matrix, $\mathcal{M}$, and the gauge interactions are all diagonal, it is possible to determine *uniquely* the heavy Majorana mass matrix, $\mathcal{M}$, and the neutrino Yukawa coupling, $\mathbf{Y}_\nu = V_R^\dagger D_Y V_L$, starting from κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. Notice that the see-saw formula, eq. (1.4), is only valid at the Majorana mass scale and not at low energies, so all the parameters in eqs. (2.4) and (2.5) should be understood at M. Therefore, the observed κ should be run from the electroweak scale to the Majorana mass scale with the corresponding RGEs[1] [12].

At this point it is worth checking that the number of physical parameters is identical at high and low energies. If we want to reconstruct the whole theory from low energy data, we must have 15 real parameters and 6 complex phases at low energies, and this is actually the case. In the basis we have chosen to work in, κ contains six real parameters and three complex phases, $\mathbf{Y_e}$ is determined by three real parameters, and P contains six real parameters and three complex phases, which add up to 15 real parameters and 6 complex phases. It is not immediately obvious that they are all independent, but we have proved that indeed they are, calculating $\mathbf{Y}_\nu$ and $\mathcal{M}$ explicitly from κ and P.

[1]This seems to require knowing the Majorana mass scale from the beginning, however, in a numerical calculation, M can be computed recursively.

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Our procedure shows that there is a one to one correspondence between the two following sets of parameters: $\{\mathbf{Y}_\nu, \mathcal{M}\}$ and $\{\kappa, P\}$, and we are free to pass from the former to the latter. A region in any of them could be mapped onto the other without loss of information. The mapping $\{\mathbf{Y}_\nu, \mathcal{M}\} \to \{\kappa, P\}$ corresponds to the common top-down approach that has been extensively used in the literature. Starting from theoretically motivated textures for $\mathbf{Y}_\nu$ and $\mathcal{M}$, one can find the corresponding κ and P and then check which of them are consistent with the neutrino data and the bounds on lepton flavour violation.

The inverse mapping, $\{\kappa, P\} \to \{\mathbf{Y}_\nu, \mathcal{M}\}$, corresponds to a bottom-up approach, which we find appealing both for theoretical and phenomenological reasons. κ and P contain the same information about the see-saw as $\mathcal{M}$ and $\mathbf{Y}_\nu$, but κ is expressed in terms of observable neutrino masses, mixing angles and phases; and P can in principle be extracted from renormalization group effects. It is therefore straightforward to restrict κ and P matrix elements to lie within their experimentally allowed ranges. The experimentally allowed $\mathcal{M}$ and $\mathbf{Y}_\nu$ can then be reconstructed. Ideally, we would like to shrink the allowed regions for κ and P to a point, and thus determine precisely $\mathcal{M}$ and $\mathbf{Y}_\nu$, but as we will see in the next section 3, this seems far from being attainable. We would like to stress here that the main goal of our paper is to find an alternative (low energy) description of the see-saw parameter space and not to determine $\mathcal{M}$ and $\mathbf{Y}_\nu$ from experiments.

This novel parametrization could be applied to the study of the cosmological baryon asymmetry generated from the out-of-equilibrium decay of the ν_Rs [8]: for certain choices of $\mathbf{Y}_\nu$ and $\mathcal{M}$, a sufficiently large lepton asymmetry is produced in the decay of the ν_Rs, and subsequently reprocessed into a baryon asymmetry by non-perturbative Standard Model B+L violating processes. Starting from κ and P, we can readily calculate the implications of experimental measurements for the leptogenesis scenario, and study the dependence of the CP violating asymmetry in ν_R decay[2] on weak scale masses and couplings [13].

To conclude this section, we would like to compare this result to the quarks, where the u_R and the u_L share a Dirac mass — there is no undetermined GUT-scale mass. The up quark masses squared are proportional to the eigenvalues of $\mathbf{Y}_u^\dagger \mathbf{Y}_u$, which is also the combination that appears in the RGEs. This is in contrast to the neutrino sector, where the light neutrino mass matrix is $\kappa = \mathbf{Y}_\nu^T \mathcal{M}^{-1} \mathbf{Y}_\nu$, and the left-handed slepton RGEs depend on $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. So, the sleptons provide additional information, complementary to the lepton mass matrices. In the neutrino sector, κ, rather than $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, is directly measurable at the weak scale. $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ contributes to the renormalization group running from the high energy scale M_X to $\mathcal{M}_i$; indeed, it is the high scale input which (in conjunction with κ) allows us to determine $\mathcal{M}$.

[2]The generated lepton asymmetry also depends on cosmological parameters, so additional assumptions are required to calculate the baryon asymmetry.

3. Present and future constraints on κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$

All the matrix elements of κ are in principle measurable, because they are determined from the light neutrino masses and the angles and phases of the MNS matrix. Three masses, three mixing angles and three phases are required to fully reconstruct κ. Atmospheric neutrino data determine the neutrino mass difference $|m_3^2 - m_2^2|$, and the mixing angle θ_{23}. Solar data allow various values for $m_2^2 - m_1^2$ and θ_{12}, although present data seem to favour the large angle MSW solution [14]. Also, reactor experiments [15] constrain θ_{13} to be small. Upcoming and proposed experiments hope to determine the solar solution, and measure the angle θ_{13}, the sign of $m_3^2 - m_2^2$, and the CP violating phase δ [16]. The overall scale of the neutrino masses could be determined from microwave background and large scale structure observations if $\sum_i m_i \gtrsim .1$ eV [17] — alternatively one could take the largest mass to be the largest mass difference. The two remaining "Majorana" phases ϕ and ϕ' contribute to $\Delta L = 2$ neutrino interactions, such as neutrinoless double beta decay [18], which could measure a combination of ϕ and ϕ'. An additional $\Delta L = 2$ process would have to be measured to determine ϕ and ϕ' separately.

We now turn to constraining P, which appears in the RGE of the left-handed slepton soft mass matrix, $\mathbf{m}_L^2$. Since P is related to radiative corrections, it is expected that the absolute value of any of its elements is $\lesssim 16\pi^2$. To further constrain P, it is necessary to make an ansatz about the high energy physics, and in what follows we make two assumptions. We use the RGEs of the MSSM with right handed neutrinos (see, for example, appendix in [11]) in running from the high scale M_X to M, and then the RGEs of the MSSM between M and M_W. This means that we neglect the effects of any other new particles between M_W and M_X, and that we are assuming $M_i \ll M_X$, so that the large log component of the ν_R-induced loops is the most significant part. Secondly, we assume that the slepton soft mass matrices at high energies are proportional to the identity. This is naturally obtained in some well motivated scenarios, for instance, minimal supergravity, dilaton-dominated SUSY breaking or gauge-mediated SUSY breaking. Also, model independent analyses of flavour changing processes suggest approximate universality of the soft terms. In making these assumptions, we are imposing some hypotheses on the high energy theory and thus our approach is not strictly bottom-up. However, these minimal assumptions are made in most top-down analyses, and are a small price to pay for parametrizing the see-saw with low energy data. With these requirements, the low energy left-handed slepton mass matrices read, in the leading-log approximation,

$$\left(m^2_{\tilde{\ell},\tilde{\nu}}\right)_{ij} \simeq (\text{diagonal part})_{\tilde{\ell},\tilde{\nu}} - \frac{1}{8\pi^2}(3m_0^2 + A_0^2)(\mathbf{Y}_\nu^\dagger)_{ik}(\mathbf{Y}_\nu)_{kj} \log\frac{M_X}{M_k}\,, \tag{3.1}$$

where "diagonal-part" includes the tree level soft mass matrix, the radiative corrections from gauge and charged lepton Yukawa interactions, and the mass contributions from F- and D-terms (that are different for charged sleptons and sneutrinos). The off-diagonal elements in eq. (3.1) induce rare lepton flavour violating processes, such as $\mu \to e\gamma$. The present upper bounds on their branching ratios can be used to constrain the absolute values of the off-diagonal elements of $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, especially $|(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{12}|$, up to a log dependence on the right-handed Majorana masses. For example, model independent analyses obtain $(m_L^2)_{12} \lesssim 20\,\mathrm{GeV}^2$ for a slepton mass of $100\,\mathrm{GeV}$ [19], which translates into $|P_{12}| \lesssim 6 \times 10^{-3}$. The observation of lepton flavour violating processes would set a lower bound on $|P_{ij}|$, $i \neq j$, for a certain choice of supersymmetric parameters.

Sparticle production at future colliders offers the possibility of further constraining the left-handed slepton mass matrices and therefore the magnitudes and phases of the elements of P. The production of sneutrinos and the observation of their decays could determine their masses and couplings to the charged leptons, which would constrain the elements of m_L^2. The diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ contribute to the running of the diagonal elements of the left-handed slepton soft mass matrix, as do the gauge and charged lepton Yukawa couplings. So elements P_{ii} that are of order the gauge couplings could be determined from the sneutrino masses.For a hierarchical $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, this appears difficult even in the case of $[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]_{33}$, as has been carefully discussed in [20]. In some areas of SUSY parameter space, slepton oscillations [21] and lepton flavour violating slepton decays [22] could determine $[m_{\tilde{\ell}}^2]_{13}$ and $[m_{\tilde{\ell}}^2]_{23}$ more accurately than rare τ decays.

The matrices $\mathbf{m}_{\tilde{\ell},\tilde{\nu}}^2$ are hermitian, so their off-diagonal elements are complex. Naively, from sneutrino oscillation experiments one could hope to extract the three phases of $\mathbf{m}_{\tilde{\nu}}^2$, however, it is likely that only one combination is measurable. In these experiments, a flavour eigenstate sneutrino could be produced with a lepton of one flavour (say e^+), oscillate among the different $\tilde{\nu}$ mass eigenstates, and then decay into a lepton of another flavour (e.g. μ^-). As discussed in [23], this process only involves one phase, which could be measured (for sneutrino mass differences of order the decay rate) in the asymmetry between the observed number of $e^+\mu^-$ and $e^-\mu^+$. The reason why there is only one phase is that the lepton number conserving[3] sneutrino mass matrix $[\mathbf{m}_{\tilde{\nu}}^2]_{ij}$ can be diagonalized, in the basis defined after eq. (1.4), by $W \mathbf{m}_{\tilde{\nu}}^2 W^\dagger = D_{\mathbf{m}^2}$. The unitary matrix W has six phases, five of which can be rotated away by choosing the relative phases of the charged leptons and sneutrinos. This removal of phases is similar to that in the CKM matrix. It is possible because the phases of the left-handed leptons are not fixed by making their mass matrix diagonal and real, so their two relative phases can be chosen to remove two phases, either in W *or* in the MNS matrix. In the case of sneutrino oscillations, two of the relative phases of the left-handed leptons used to redefine W will reappear in the MNS matrix, but this does not have any physical effect since this experiment does not involve neutrinos.

[3] We neglect for the moment the sneutrino "Majorana" mass matrix, appearing in $[m_{\tilde{\nu}\tilde{\nu}}^2]_{ij} \tilde{\nu}_i \tilde{\nu}_j +$ h.c.

Thus, to measure the remaining two phases we need an experiment probing the sneutrino-neutrino-neutralino vertex, that depends on both W and the MNS matrix. Experiments that involve the MNS matrix are difficult to perform, so measuring these two phases does not seem feasible. An alternative approach might be to look for CP violation in sneutrino-anti-sneutrino oscillations [24]. Sneutrinos can oscillate into anti-sneutrinos due to small $\Delta L = 2$ sneutrino masses $[m^2_{\tilde{\nu}\tilde{\nu}}]_{ij}\tilde{\nu}_i\tilde{\nu}_j + $h.c., which are the soft SUSY breaking analogy of the neutrino Majorana masses: $[m^2_{\tilde{\nu}\tilde{\nu}}]_{ij} \sim v_2^2 m_0 \kappa_{ij}$. However, this process is only observable in a small area of the SUSY parameter space [24], so it is unlikely that the phases could be extracted from these phenomena.

4. Discussion

In the best of all supersymmetric worlds, where soft terms are universal at the GUT scale, and all masses, couplings and phases have been measured at the weak scale, it is possible to calculate the GUT-scale inputs of the SUSY see-saw from measured quantities. However, in practice it would be very difficult to make all the weak scale measurements required to determine the neutrino Yukawa matrix, $\mathbf{Y}_\nu$, and the right-handed neutrino Majorana mass matrix, $\mathcal{M}$. The magnitude of the off-diagonal elements of $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ induce lepton flavour violation in the left-handed slepton mass matrices, so could be measurable. The diagonal elements of $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ are more difficult: they modify the diagonal elements of the left-handed slepton soft mass matrix, so they would be hard to disentangle from gauge and charged lepton Yukawa contributions to the RG running. Finally, most of the CP violating phases in $\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu$ and the MNS matrix seem practically unattainable.

A less ambitious, and at this stage more practical, interpretation of our results, is that it is possible to describe the see-saw parameter space using just inputs at the electroweak scale, which have a very straightforward physical interpretation (neutrino masses and mixings, rates for rare flavour changing processes, sneutrino masses ...). Since we make our experiments at the electroweak scale, it is much more natural to use a parameter space spanned by quantities measurable at the electroweak scale rather than at the GUT scale. This approach has the immediate consequence that the see-saw parameter space is already constrained by neutrino data and rare lepton decays, and might be further constrained in the future with the advent of neutrino factories and LHC. Therefore, model independent conclusions about the seesaw mechanism can be drawn more readily in this bottom-up formulation than in a top-down approach, where a priori the parameter space is not constrained at all!

Our parametrization is very convenient to look for ways to test the see-saw mechanism. We have shown that it is difficult to rule out the see-saw mechanism from neutrino data and radiative effects on left-handed slepton soft mass matrices: for *any* combination of low energy neutrino parameters (encoded in κ) and left-handed slepton mass matrices (that can be parametrized by P, as in eq. (3.1)), it is *always* possible to find a neutrino Yukawa coupling and a Majorana mass matrix that reproduce those low energy observables.[4] If those $\mathbf{Y}_\nu$ and $\mathcal{M}$ are not compatible with perturbativity or the experimental lower bounds on Majorana masses, it would be a blow against the see-saw. Also, the observation of lepton flavour violation not encoded in κ and P (for instance from the right-handed slepton mass matrix), would be an indication for physics other than the see-saw with universal soft terms. In this parametrization, neutrino masses and mixing angles, and radiative effects on slepton masses, are inputs, and in this sense cannot be regarded as predictions of the see-saw mechanism. Therefore, to test the see-saw, we need another low energy effect which is a consequence of it, such as the baryon asymmetry of the Universe. The CP asymmetry in right-handed neutrino decay is indeed a prediction of the see-saw and, when combined with assumptions about the early evolution of the Universe, could be compared with the cosmological baryon asymmetry.

In summary, we have shown that in a SUSY model with universal soft terms, it is possible to determine the GUT scale inputs of the see-saw from parameters that are in some sense measurable at the weak scale. The right-handed neutrino Majorana mass matrix, $\mathcal{M}$, and the neutrino Yukawa matrix, $\mathbf{Y}_\nu$, can be calculated from the light neutrino masses, the MNS matrix, and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ (which enters into the renormalization group equations of the left-handed slepton soft mass matrix). This does not conflict with decoupling theorems because $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ (whose indices are left-handed) is effectively a high-scale input: it contributes to the slepton mass matrix via renormalization group running at scales above M. This matrix can be extracted from $m_{\tilde{\nu}}^2$ if the soft masses are universal at the high scale M_X, and if the main contribution to the RGEs below M_X is from the MSSM particles and the right-handed neutrinos. We briefly discussed the (not very encouraging) prospects of measuring the magnitudes and phases of all κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ matrix elements, in a model with universal soft terms. A more realistic approach is to impose all the available experimental constraints on κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, vary the matrix elements over their experimentally allowed ranges, and calculate the resulting allowed values of $\mathbf{Y}_\nu$ and $\mathcal{M}$. We will pursue this bottom-up approach to the see-saw (and its application to leptogenesis) in subsequent work [13].

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[4]Notice that this is not obvious in the top-down approach: for example, it is not immediately clear that there is a region of the $\mathbf{Y}_\nu$ and $\mathcal{M}$ parameter space consistent with any experimentally determined neutrino parameters and with the upper bounds on rare lepton decays.

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